Simple Analytic Expressions for the Total Cross Section for γ-e Pair Production

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The total cross section for pair production by photons in the field of free electrons is fitted to simple analytic expressions in four intervals of the incident photon energy which cover the whole range between threshold and infinity.

The knowledge of accurate cross sections for pair production in the field of electrons or positrons (triplet production) is required in evaluations of total photon absorption cross sections [1, 2], for calculations of the cross section for pair production in the field of atoms [3], and in astrophysical problems [4–6]. Compared to the familiar Bethe-Heitler formulae [7] for pair production in the Coulomb field of a nucleus, the cross section for triplet production, though obtained by straightforward application of quantum electrodynamics, is extremely complicated due to recoil and exchange effects [8]. Haug [9] has succeeded in deriving an expression for the doubly differential cross section for pair production in the field of free electrons in lowest-order perturbation theory (Born approximation) without neglecting the Feynman diagrams which describe exchange and the interaction of the incident photon with the initial electron. This lengthy formula has to be integrated numerically for obtaining the total cross section σ_t. In order to facilitate the calculation of σ_t, the total cross section for triplet production has been fitted to simple analytical expressions in four intervals of the photon energy, the fractional error of the approximation being less than 0.3% everywhere, mostly less than 0.1%.

The total cross section σ_t is a relativistic invariant; it is a function of the invariant product of the energy-momentum four-vectors of the incident photon and electron which in the rest system of the target electron reduces to the photon energy k. If k is given in units of the electron rest energy, mc^2, the threshold for triplet production is $k = 4$.

In the low-energy region $4 \leqq k \leqq 4.6$, the cross section derived by Votruba [10] has been used as the leading term of a power series in $(k-4)$. Defining

$$f(k) = \sigma_t(k)/\alpha r_0^2\,, \tag{1}$$

where $\alpha \approx 1/137$ is the fine-structure constant and $r_0^2 = e^2/mc^2$ the classical electron radius, $f(k)$ has the form

$$f(k) = \{5.6 + 20.4(k-4) - 10.9(k-4)^2 - 3.6(k-4)^3 + 7.4(k-4)^4\} \cdot 10^{-3}(k-4)^2\,, \quad 4 \leqq k \leqq 4.6\,. \tag{2}$$

In the range $4.6 \leqq k \leqq 6.0$, $f(k)$ can be fitted by a simple power series in k:

$$f(k) = 0.582814 - 0.29842\,k + 0.04354\,k^2 - 0.0012977\,k^3\,, \quad 4.6 \leqq k \leqq 6.0\,. \tag{3}$$

The best approximation for medium energies is given by

$$f(k) = \frac{3.1247 - 1.3394\,k + 0.14612\,k^2}{1 + 0.4648\,k + 0.016683\,k^2}\,, \quad 6 \leqq k \leqq 18\,. \tag{4}$$

In the high-energy region the series is based on the three leading terms of the Borsellino formula [8, 11]:

$$f(k) = \frac{28}{9}\ln(2k) - \frac{218}{27} + \frac{1}{k} \cdot \{-\tfrac{4}{3}(\ln 2k)^3 + 3.863(\ln 2k)^2 - 11\ln(2k) + 27.9\}\,, \quad k \geqq 14\,. \tag{5}$$

The latter expression is less complicated and more accurate than the Borsellino-Ghizzetti formula [12]. The fractional errors compared to the numerically computed cross sections [9] are $< 0.3\%$ for $14 \leqq k \leqq 30$ and $< 0.1\%$ for $k \geqq 30$.

The analytic shape of the approximations (2) to (5) is chosen such that they can be integrated in closed form over a variety of distribution functions for the photon energy k.

The present empirical formulae for the triplet cross section complement the series expansion derived by Maximon [13] for the total cross section of photon pair production in the Coulomb field of nuclei.

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